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Intermediate $E(k, L)$ statistics in the regime of mixed classical dynamics

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Abstract. We explain the arguments in support of the Berry–Robnik (1984 *J. Phys. A: Math. Gen.* **17** 2413) picture of the energy-level statistics in the asymptotic strict (far) semiclassical limit of sufficiently small $\bar{h}$, where the entire energy spectrum can be represented as a statistically independent superposition of regular and irregular level sequences, the regular ones obeying the Poissonian statistics and the irregular ones the RMT statistics (GOE or GUE). We generalize the results to describe not only the level spacing distribution, the number variance and the delta statistics, but also arbitrary $E(k, L)$ statistics (= the probability to find k levels inside an interval of length L—after unfolding). Very useful and effective approximations for $E(k, L)$ are described. We demonstrate very clearly that this regime is excellently described by this picture for k as high as $L_{\max}$, the outer energy scale, even when taking into account the regular component and only one (the dominant) chaotic component. This we show numerically for the compactified standard map and for the quartic 2D billiard.

The generic classical Hamiltonian dynamical systems $H(\boldsymbol{q}, \boldsymbol{p})$ with only a few (N) degrees of freedom are neither integrable nor completely chaotic (ergodic), but are somewhere in between, for they have a divided phase space comprised of regular regions (covered by invariant tori) and chaotic regions (where the classical orbits possess positive largest Lyapunov exponents and are dense inside). The picture is very well described by the KAM Theory. We still do not have a proof, even for a single nontrivial dynamical system, that the invariant measure of the chaotic component is indeed positive—the so-called *coexistence problem* (Strelcyn 1991), although in physics we have little doubt that this is true. Recently we have discussed this and related problems and have shown how to calculate, also numerically, the measure of the chaotic component (Prosen and Robnik 1998).

We do have strong evidence that classically integrable systems exhibit Poissonian spectral statistics (Robnik and Veble 1998), and that the classically ergodic systems have statistical properties of random matrix theory (RMT), either GOE or GUE, depending on whether the system has or does not have an antiunitary symmetry, respectively. The latter statement is known as the Bohigas conjecture (Bohigas *et al* 1984). An excellent review covering a wide spectrum of closely related problems in quantum chaos has been recently published by Weidenmüller and coworkers (Guhr *et al* 1998). The above classes are the so-called *universality*

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classes (of spectral fluctuations), as their behaviour does not contain any free parameter. They emerge, as a phenomenon, only after eliminating the specific features of a system under study, above all after performing the unfolding procedure. This is done by reducing the spectrum in such a way that the mean level spacing is equal to one everywhere, which can always be done, in the leading order, by using the Thomas–Fermi rule for the mean density of energy levels†.

The question of the spectral statistics of intermediate regimes has been addressed for the first time in (Robnik 1984), where a continuous transition in a KAM-type billiard (Robnik 1983) from Poisson to GOE has been demonstrated numerically with increasing deformation of the circle billiard. Just a few months later the theory of spectral statistics in such an intermedite regime was published by Berry and Robnik (1984), derived by analysing the level spacing distribution $P(S)$. (Qualitatively, the idea goes back to the work of Percival (1973).) This picture rests upon the assumption of the statistically independent superposition of Poissonian sequences, associated with the regular regions (invariant tori), labelled by $j = 1$, and the GOE/GUE sequences, associated with the chaotic regions, labelled by $j = 2, 3, \ldots, m$. It was realized already in that work that the nice and most convenient mathematical object for the level spacing analysis is the gap probability $E(0, L)$, by definition equal to the probability that in an interval of length L there is no level. We have $P(S) = \mathrm{d}^2E(0, S)/\mathrm{d}S^2$ (see below and Haake 1991, Mehta 1991 and Aurich *et al* 1997). Namely, it is the gap probability that factorizes under the statistically independent superposition of level sequences (see below). Later an analogous analysis was performed by Seligman and Verbaarschot (1985) for the sigma statistics (number variance) and the delta statistics, which have the additivity property. Recently, it has been emphasized by Steiner and coworkers (Aurich *et al* 1997) that the $E(k, L)$ statistics are not only quite fundamental but also very convenient in the numerical work, in general. By definition, $E(k, L)$ is the probability of finding precisely k levels in an interval of length L (after unfolding). We shall see below that they are indeed most natural statistical measures to analyse in the mixed systems. Moreover, $P(S)$ and the number statistics and thus also the delta statistics can be simply expressed in terms of $E(k, L)$ statistics (Robnik 1998, Aurich *et al* 1997).

The Berry–Robnik picture is based on the so-called principle of uniform semiclassical condensation, which states that the Wigner functions of the stationary eigenstates in the strict semiclassical limit $\hbar \to 0$ become uniformly extended on the classical invariant objects supporting the quantal eigenstates, as implicit in Berry (1977, 1983) and Robnik (1988). A recent review was published by Robnik (1998), and a compact comment regarding this topics was published in (Robnik and Prosen 1997). The principle indeed implies 'no interaction between different level sequences' and thus their statistical independence. The approach to this asymptotic regime in relation to localization phenomena of chaotic eigenstates and to the implied phenomenon of fractional power-law level repulsion has been studied in (Prosen and Robnik 1993, 1994a, b).

Thus, it is important to know *the relative invariant (Liouville) measure* of chaotic and regular eigenstates because the Hilbert space of a mixed Hamiltonian system is split into regular and irregular eigenstates, in the strict semiclassical limit, precisely in proportion to the classical invariant measure of the integrable component (invariant tori) and of the irregular components.

† There might very well be an additional, so-far overlooked universality class of semi-Poissonian statistics, recently discussed by Bogomolny *et al* (1998), which, seems to appear in the systems which contain chaotic orbits dense in the phase space but not having invariant measure one (i.e. they are topologically transitive but not metrically transitive and thus non-ergodic). One example of such systems is the rational polygon billiard, which is pseudo-integrable, having invariant surfaces of genus higher than one (Berry 1983, Richens and Berry 1981).

The invariant Liouville measure of a subset ω of the energy surface is equal to

$$\rho(\omega) = \frac{\int \mathrm{d}^N \boldsymbol{q}\, \mathrm{d}^N \boldsymbol{p}\, \delta_1(E - H(\boldsymbol{q}, \boldsymbol{p}))\chi_\omega(\boldsymbol{q}, \boldsymbol{p})}{\int \mathrm{d}^N \boldsymbol{q}\, \mathrm{d}^N \boldsymbol{p}\, \delta_1(E - H(\boldsymbol{q}, \boldsymbol{p}))} \tag{1}$$

where $\chi_\omega(\boldsymbol{q}, \boldsymbol{p})$ is the characteristic function on the set ω, $\delta_1(x)$ is the one-dimensional Dirac delta function, N is the number of degrees of freedom, E is the energy and $H(\boldsymbol{q}, \boldsymbol{p})$ is the Hamiltonian. The relative invariant Liouville measure of the regular components will be denoted by ρ_1, and the measures of chaotic components (ordered in sequence of decreasing measure) by $\rho_2, \rho_3, \ldots, \rho_m$, where $m = \infty$ for $N = 2$ and $m = 2$ for $N \geqslant 3$, since in case of three or more degrees of freedom we have the phenomenon of the Arnold web, which is dense in the phase space, and thus there is, strictly speaking, only one (but very complex) chaotic component on the energy surface. In (Prosen and Robnik 1998) we have explained how to calculate $\rho_1, \rho_2, \rho_3, \ldots$. Of course by the assumption of normalization (of the relative measure and of the level density) we have $\rho_1 + \rho_2 + \cdots + \rho_m = 1$.

Assuming the above-mentioned absence of correlations pairwise between m spectral sequences, due to the fact that they have disjoint supports and thus do not interact, where m is infinite for $N = 2$ and 2 for $N \geqslant 3$, the spectral statistics can be written as (Robnik 1998)

$$E_{\mathrm{mixed}}(k, L) = \sum_{k_1+k_2+\ldots+k_m=k} \prod_{j=1}^{m} E_j(k_j, \rho_j L) \tag{2}$$

which is a most general manifestation of Berry–Robnik (1984) picture. Here $E_j(k, L)$ is $E_{\mathrm{Poisson}}(k, L)$ for $j = 1$, and $E_{\mathrm{RMT}}(k, L)$ for $j = 2, 3, \ldots, m$. The former is equal to

$$E_{\mathrm{Poisson}}(k, L) = \frac{L^k}{k!} \exp(-L) \tag{3}$$

whilst the latter can be found in Mehta (1991), namely in the form of numerical tables for $k = 0$–7 and in (Aurich *et al* 1997) in the form of asymptotic Gaussian formulae for $k \geqslant 8$, namely

$$E_{\mathrm{GOE}}(k, L) \approx \frac{1}{\sqrt{2\pi\alpha(L)}} \exp\left(-\frac{(L-k)^2}{2\alpha(L)}\right) \tag{4}$$

where $\alpha(L) = \Sigma^2_{\mathrm{GOE}}(L)$, which is precisely the number variance (number statistics). In fact, this same Gaussian asymptotic formula applies also to the Poissonian case (3), where we must use $\alpha(L) = L$ instead.

The picture is based on the reasonable assumption that (after unfolding) the mean density of levels in the jth sequence of levels is ρ_j, simply applying the Thomas–Fermi rule of filling the phase space volume with elementary cells of size $(2\pi\hbar)^N$ in the thin energy shell embedding the corresponding subset ω. Therefore, please note that the second argument of $E_j(k, L)$ is weighted precisely by the classical relative invariant measure of the underlying invariant component. Also, if there are several regular (Poissonian) sequences they can be lumped together into a single Poissonian sequence (which we traditionally label by 1 with relative invariant measure ρ_1). It is easy to show (Robnik 1998), that if $\alpha_1, \alpha_2, \ldots, \alpha_l$ are positive real numbers and β being their sum, $\beta = \alpha_1 + \alpha_2 + \cdots + \alpha_l$, then for all k, and L,

$$E_{\mathrm{Poisson}}(k, \beta L) = \sum_{k_1+k_2+\cdots+k_l=k} E_{\mathrm{Poisson}}(k_1, \alpha_1 L) E_{\mathrm{Poisson}}(k_2, \alpha_2 L) \ldots E_{\mathrm{Poisson}}(k_l, \alpha_l L) \tag{5}$$

by simply using the definition of $E_{\mathrm{Poisson}}(k, L)$ of equation (3). Thus, we have some kind of a central limit theorem, saying that the statistically independent superposition of Poisson sequences results in a Poisson sequence, such that the total density of Poissonian levels β is equal to the sum of the partial level densities α_j, $j = 1, 2, \ldots, l$.

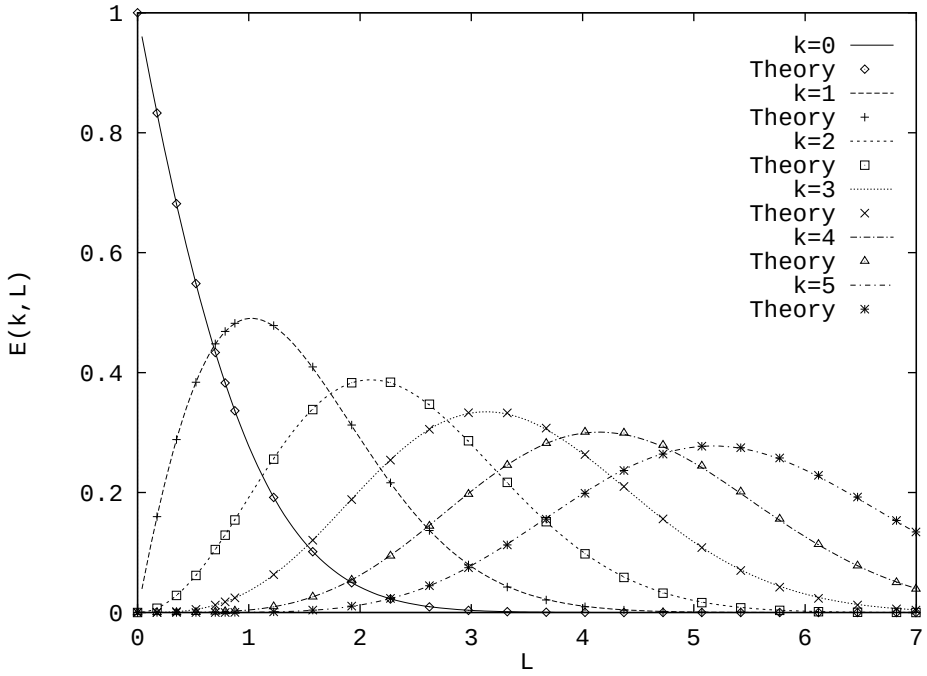

Figure 1. $E(k, L)$ statistics, $0 \leqslant k \leqslant 5$, of the compactified standard map for parameter $a = 1.8$. Numerical statistics, obtained as averages over 20 spectral samples ($m = 7991, \ldots, 8000$, both parities) of $m \approx 8000$ levels each, are shown as continuous curves (see legend), while theoretical results for the classical parameter $\rho_1 = 0.27$ (based on the table of $E_{\mathrm{GOE}}(k, L)$ from Mehta (1991)) are shown as point symbols.

Now we shall apply the general theory (2) for three two-dimensional systems ($N = 2$), namely the compactified standard map (Prosen and Robnik 1994a, b) with the kick parameter $a = 1.8$ and $a = 1.3$, and for the quartic billiard (Prosen 1998) with $a = 0.04$, and in all cases we shall take into account *only one chaotic component*, labelled by $j = 2$. It is quite typical in strongly pronounced KAM-type regimes that the dominant (largest) chaotic component is by orders of magnitude, say factor 100, larger than the next (subdominant) one. Thus, in such cases we expect that this approximation will lead to a good description of the level statistics of the mixed system. Indeed, this is confirmed excellently in our results, described below. We assume $\rho_1 + \rho_2 = 1$. For the details of the definition of these model systems see (Prosen and Robnik 1994a, b, Prosen 1998).

For $m = 2$ we get the simple formula

$$E_{\mathrm{mixed}}(k, L) = \sum_{l=0}^{k} E_{\mathrm{Poisson}}(l, \rho_1 L) E_{\mathrm{RMT}}(k - l, \rho_2 L) \tag{6}$$

where the Poissonian $E_{\mathrm{Poisson}}(k, L)$ is known exactly in (3) whilst $E_{\mathrm{RMT}}(k, L)$ is taken from the book of Mehta (1991) (numerical tables for $k \leqslant 7$) and its asymptotic formulae for $k \geqslant 8$ are given in (4). Also, it should be noticed that if we use the asymptotic formulae (4) for sufficiently large k, for both components $j = 1$ and $j = 2$, then the final result, after replacing the discrete convolution (6) by a continuous one (summation over $k \rightarrow$ integration over k), is

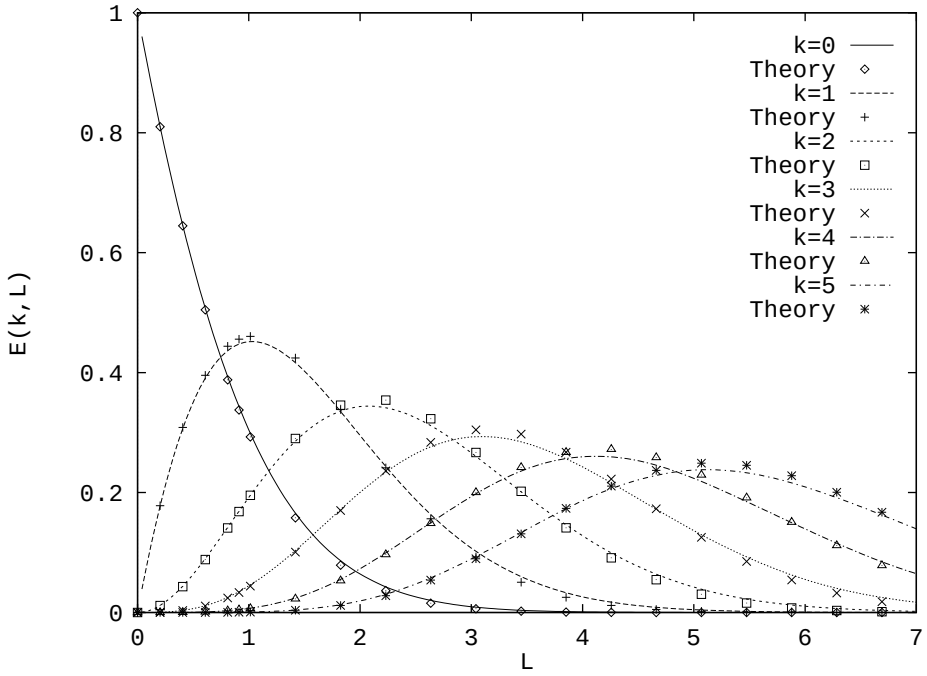

Figure 2. As figure 1, but for $a = 1.3$ and $\rho_1 = 0.37$.

again a Gaussian:

$$E_{\text{mixed}}(k, L) \approx \frac{1}{\sqrt{2\pi\gamma(L)}} \exp\left(-\frac{(L-k)^2}{2\gamma(L)}\right) \tag{7}$$

where $\gamma(L) = \alpha_1(\rho_1 L) + \alpha_2(\rho_2 L)$, and $\alpha_1(L)$ is the number variance of the Poissonian sequence $j = 1$ and $\alpha_2(L)$ is the number variance of the chaotic sequence $j = 2$, so that $\gamma(L)$ is the number variance of the total spectrum.

In figure 1 we show the results for the compactified standard map at $a = 1.8$. Each spectral stretch has 8000 levels, and we have averaged over 20 such stretches, so that in the statistics about 160 000 objects were used. The precise theoretical value has been calculated (at discrete points) using the above approach (Mehta 1991) and it is clearly seen that the agreement is just perfect within the graphical resolution. Here we used as the input parameter *the classical value of* $\rho_1 = 0.27$, rather than making the (nonlinear) least-square fit and comparing the extracted quantal value with the classical one. Of course, we also looked at the sensitivity of this graph with respect to the value of ρ_1, by changing it up and down by 0.01, resulting in a small disagreement (deviation) of about 3%. In figure 2 we show the same quantities for kick parameter equal to $a = 1.3$ and $\rho_1 = 0.37$. Here a small disagreement is seen, which can be understood not necessarily in terms of the wrong ρ_1, but in terms of not being far enough in the semiclassical limit see (Prosen and Robnik 1994a, b). Nevertheless, the agreement can still be considered as very good.

In figure 3 we show the same as in figure 1, but now for higher values of $k = 15$–20. The agreement is excellent, even when using the asymptotic formulae (4). In figure 4 we show the same objects as in figure 3, but now in the (semi)logarithmic scale, showing very clearly that the agreement is excellent *even in the tails* of $E(k, L)$ where its value is small and one would expect the error to become comparable to the value of the function itself. In figure 5 we show

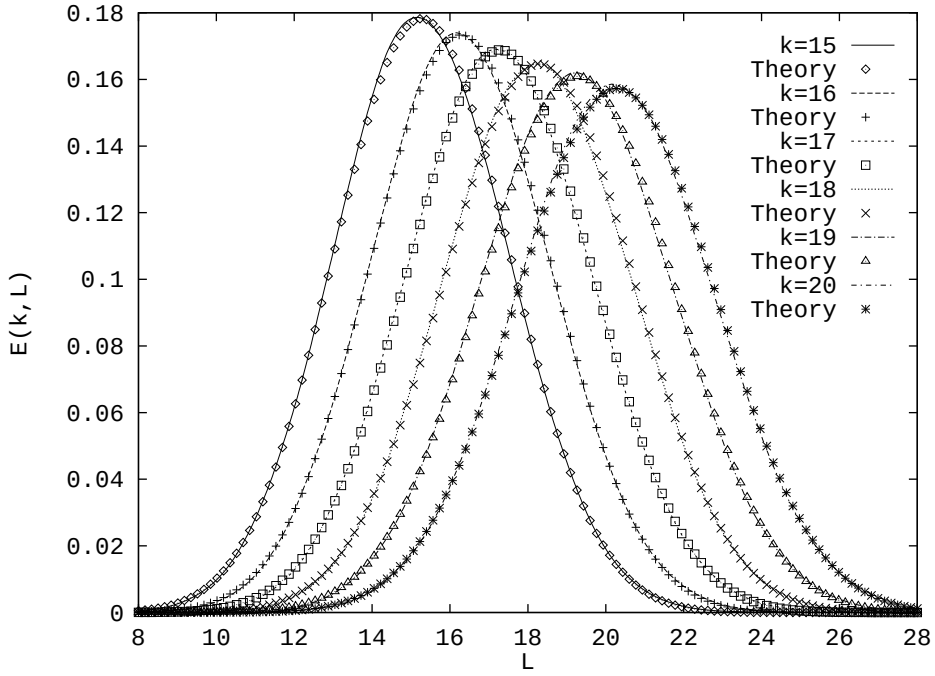

Figure 3. As figure 1, but for $15 \leqslant k \leqslant 20$.

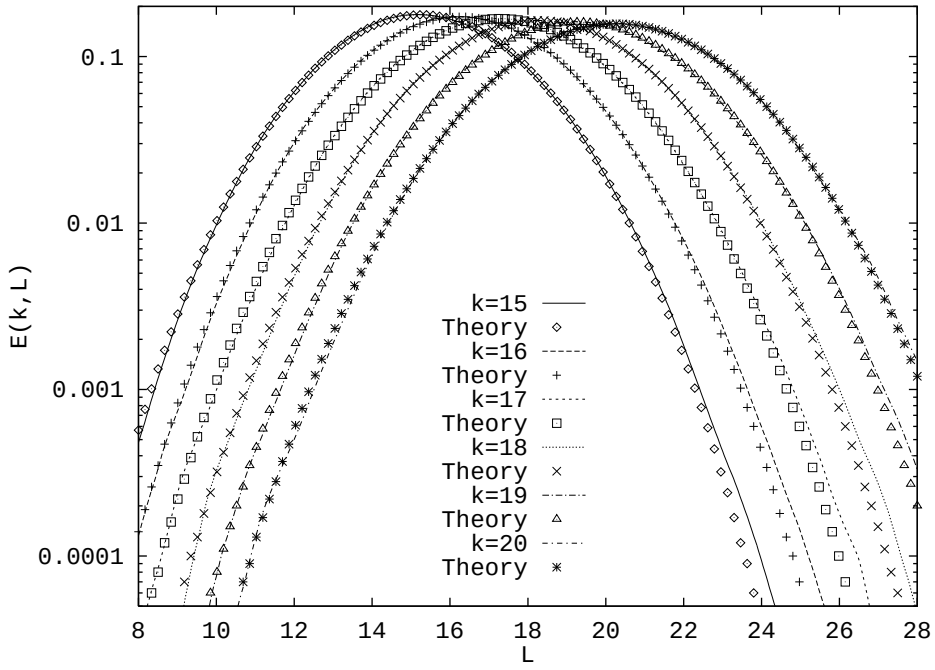

Figure 4. As figure 3, but in semi-logarithmic scale in order to exhibit the tails.

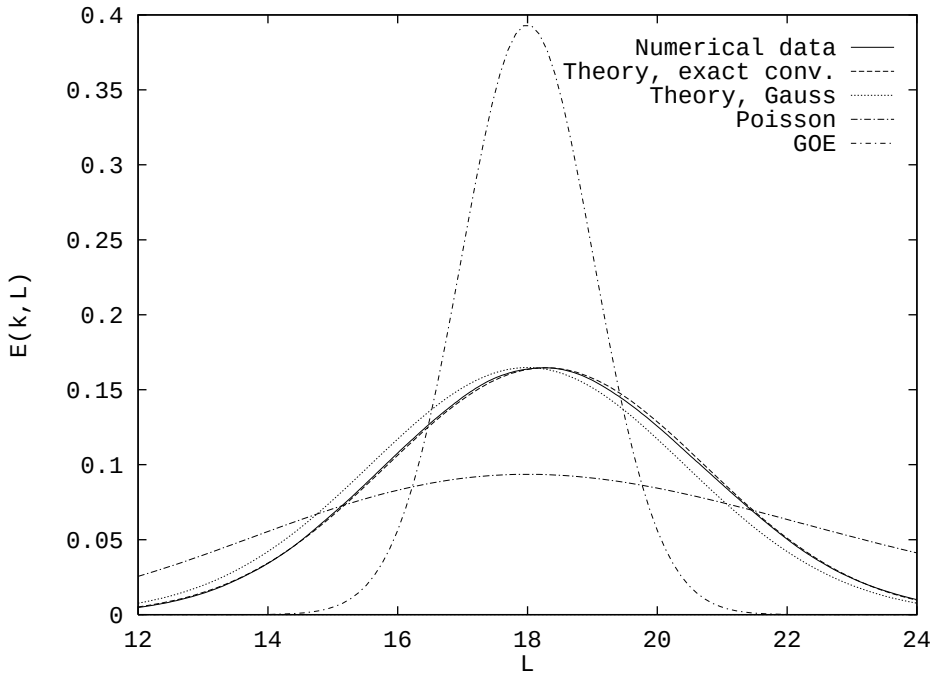

Figure 5. $E(k = 18, L)$ for the compactified standard map at $a = 1.8$ (full curve) is compared with the limiting Poisson (short chain curve) and GOE (long chain curve) statistics, and with the two theoretical curves for $\rho_1 = 0.27$, namely with 'exact' theory (equation (6)) (broken curve), and with Gauss approximation (equation (7)) (dotted curve).

$E(k = 18, L)$ for the same system, with the kick parameter $a = 1.8$, compared with the exact theory, the Gaussian approximations (4), the Poisson formula (3) and the GOE result (4).

With increasing k and L we, of course, expect larger deviations from the theory, for two reasons: we approach the finite size of our spectral stretches and also the outer energy scale, $L_{\max} = \bar{h}/(T_0 \Delta E)$, discovered by Casati *et al* (1985) and Berry (1985), and fully determined by the shortest classical periodic orbit in the system with period T_0 and the mean level spacing equal to ΔE. Indeed, this can be observed in figure 6, where we plot the same quantities as in figure 1 but now for $k = 88, 92, 96$ and $k = 100$. The largest deviations are near the peaks of $E(k, L)$ curves.

In order to clearly expound these trends we went even further, up to $k = 1000$, where we expect nontrivial structure of $E(k, L)$ as functions of L, because we have now already entered the *nonuniversal regime* with $L \geqslant L_{\max} \approx 1000$ (see below), as shown in figure 7.

In figures 8 and 9 we show the same objects as in figure 1 but now for 5168 consecutive levels of the quartic billiard defined and introduced in Prosen (1998), with $a = 0.04$. The value of $\rho_1 = 0.12$ has been used. The agreement is excellent in figure 8 and very good in figure 9, but not as good as in the standard map (figures 1 and 3), which is also due to the statistically smaller spectral samples.

At this point we must comment on the reasons for deviation of the actual $E(k, L)$ from the semiclassical one, (2), and more specifically (6). The main reason is that at sufficiently large L, beyond the outer energy scale, we reach *the nonuniversal regime*, as was discovered by Casati *et al* (1985) and explained by Berry (1985) for the sigma and delta statistics. We expect exactly the same characteristic energy scale $L_{\max}$ to control the behaviour of $E(k, L)$,

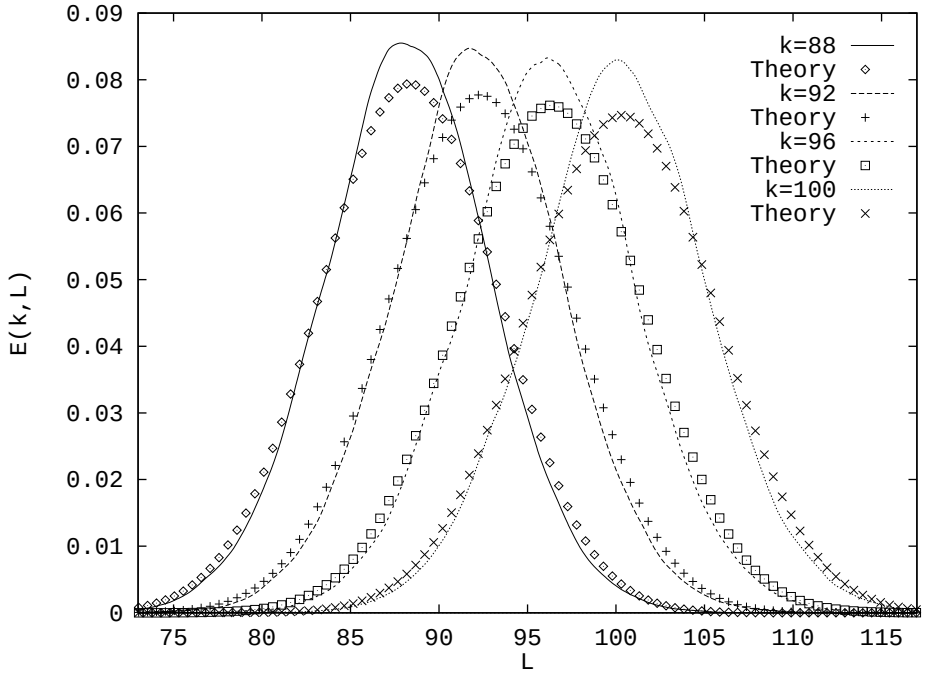

Figure 6. As figure 1, but for $k = 88, 92, 96, 100$.

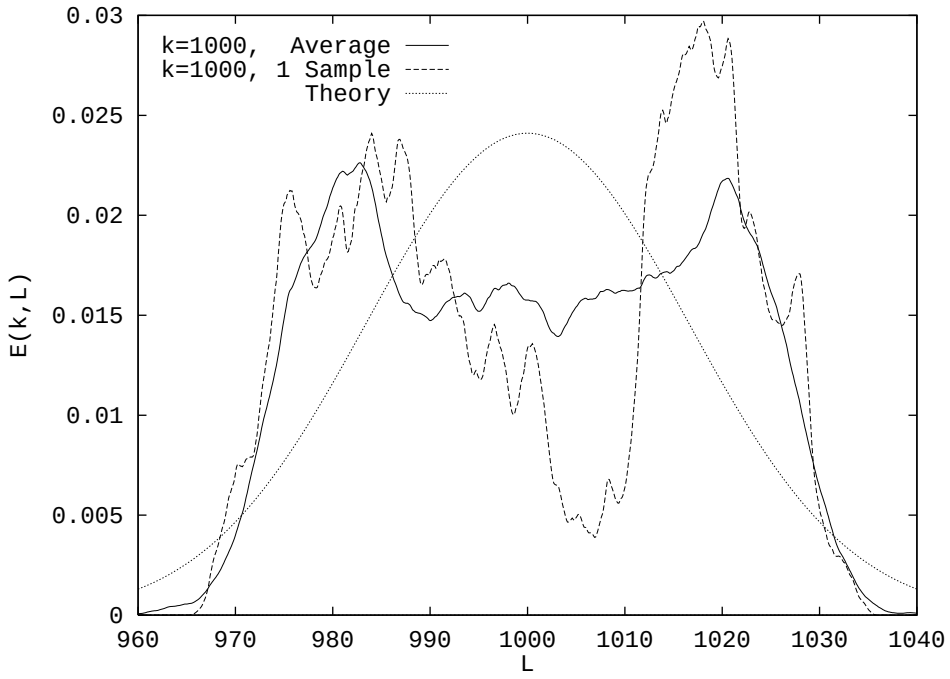

Figure 7. $E(k = 1000, L)$ statistic of the compactified standard map at $a = 1.8$ in *nonuniversal* regime $k \geqslant L_{\max}$. The full curve gives the average over 20 spectral samples ($m = 7991 \ldots 8000$, both parities), while the broken curve gives the statistic for a single spectral sample ($m = 8000$, even parity). For comparison we plot the theoretical distribution (dotted curve).

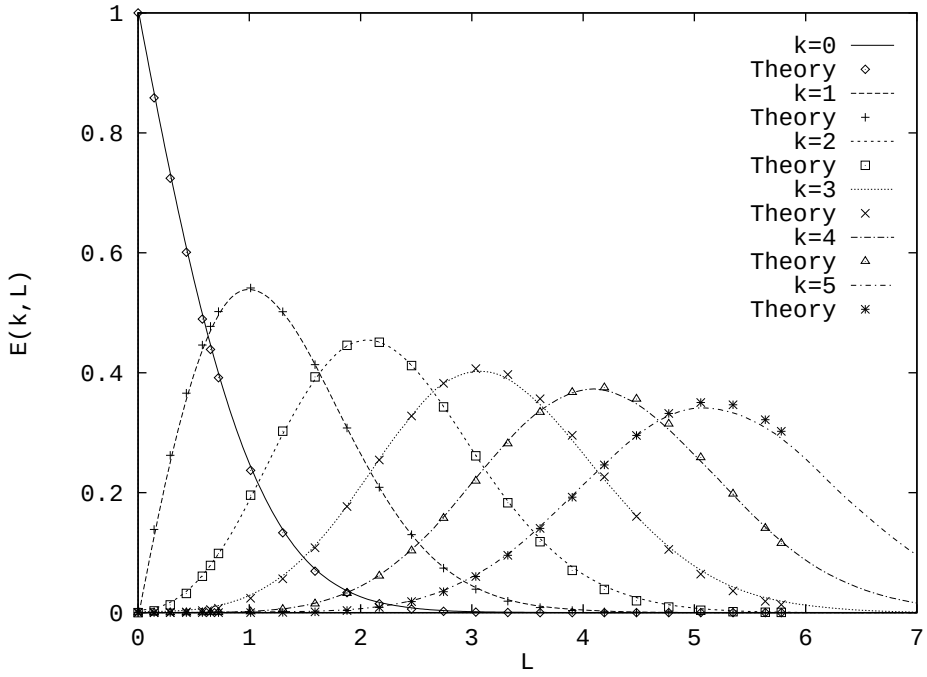

Figure 8. As figure 1, but for 5168 consecutive levels of the quartic billiard (Prosen 1998) for $a = 0.04$ with sequential quantum number $\mathcal{N} \approx 8\,000\,000$, and for theoretical distributions with $\rho_1 = 0.12$.

as could be shown by a systematic employment of Gutzwiller trace formula (Gutzwiller 1990 and references therein, also Berry 1985), but this question of course transcends the scope and possibilities in this paper, and thus remains as an interesting, important and challenging new project. Beyond the outer energy scale, $L > L_{\max}$, the behaviour of sigma, delta and other statistics, including $E(k, L)$, as functions of L, cannot be expected to behave universally, and therefore we cannot expect the Gaussian approximation to be good, either for individual subsequences or for the entire spectrum. Thus, the 'wild' behaviour of $E(k, L)$ as functions of L in such a nonuniversal regime as, e.g., shown in figure 7, and also the deviations clearly seen in figure 9 for the quartic billiard, is precisely due to these complications. However, qualitatively we should explain the following. In the case of statistically indepedendent superposition of spectra, labelled by $j = 1, 2, \ldots m$, we find the deviation from universality for each individual subsequence of levels for $L > L^{(j)}_{\max}$, $j = 1, 2, \ldots, m$, where $L^{(j)}_{\max}$ is associated with individual spectral subsequence labelled by j, and therefore is equal to $L^{(j)}_{\max} = \overline{h}\rho_j/(T^{(j)}_0 \Delta E)$ where ρ_j is the density of levels of the jth subsequence, and $T^{(j)}_0$ is the period of the shortest classical periodic orbit in the jth classical invariant component. Of course, each $T^{(j)}_0$ certainly cannot be smaller than T_0; in fact, it would typically be larger than T_0. Also, ρ_j is always smaller than 1; in fact it can be very small. In our case of the quartic billiard mentioned above $\rho_1 = 0.12$, which means that $L^{(1)}_{\max} \approx 300$ rather than $L_{\max} \approx 3000$, and for the standard map ($a = 1.8$) it is $L^{(1)}_{\max} \approx \rho_1 L_{\max} \approx 1000$ rather than the maximal outer pseudo-energy scale which, for the m-dimensional quantum map, is estimated simply as $L_{\max} \approx m/2 = 4000$ due to the periodicity of the quasi-energy levels. It is, however, true that at smaller ρ_j, when it goes to zero, the contribution of the jth subsequence to the total spectrum becomes less and less

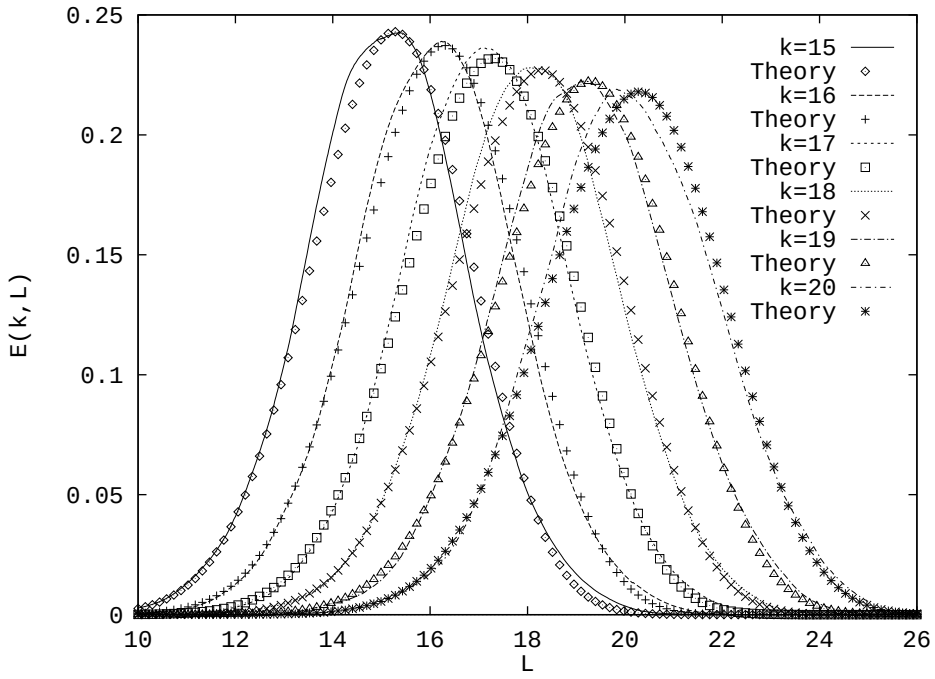

Figure 9. As figure 8, but for $15 \leqslant k \leqslant 20$.

important, and eventually irrelevant, whatever the value of $L_{\max}^{(j)}$. But it is hard, at this stage, to say more about this nonuniversality regime.

We may conclude that the Berry–Robnik (1984) picture has been fully confirmed not only for $P(S)$ and $\Delta(L)$ but in fact for all $E(k, L)$ statistics in the three dynamical systems that we study numerically in this paper. The theoretical arguments have been explained in detail in the recent review (Robnik 1998), where the reasons for the correctness of this picture are given, and also the phenomena of the deviation from the asymptotic behaviour are described (localization of chaotic states and the fractional power-law level repulsion) for when we are not deep enough in the semiclassical regime (not sufficiently small $\overline{h}$). The qualitative criterion for the crossover regime (from Brody-like to Berry–Robnik) consists in comparing the classical diffusion timescale (ergodic time) and the break time (Heisenberg time). If the latter is much shorter than the former, localization (of chaotic states) appears and manifests itself in the fractional power-law level repulsion and thus Brody-like behaviour. This important regime has also been recently studied by Engel *et al* (1998) also by analysing the higher-order level spacing statistics. But the ultimate asymptotic semiclassical limiting behaviour is definitely correctly described by the Berry–Robnik picture, which is the main conclusion of this paper.

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